# RAPTOR equation summary

F. Felici, f.felici@tue.nl

November 29, 2012

## 1 Definitions

$V' = \frac{\partial V}{\partial \rho}$, $G_1 = \left\langle (\nabla \rho)^2 \right\rangle$, $G_2 = \frac{V'}{4\pi^2} \left\langle \frac{(\nabla \rho)^2}{R^2} \right\rangle$.

$g_1 = \left\langle (\nabla V)^2 \right\rangle$, $g_2 = \left\langle \frac{(\nabla V)^2}{R^2} \right\rangle$, $g_3 = \left\langle \frac{1}{R^2} \right\rangle$.

$\rho_e = \sqrt{\Phi_b/\pi B_0}$, $F = RB_\phi$

$j_{ni} = \frac{\langle \mathbf{j_{ni}} \cdot \mathbf{B} \rangle}{B_0}$

## 2 Flux transport

Flux surface averaged ohm's law in general form:

$$\sigma_\parallel \left. \frac{\partial \Phi}{\partial t} \right|_\psi = \frac{F^2}{2\pi\mu_0} \frac{\partial}{\partial \psi} \left[ \frac{g_2}{F} \frac{\partial \psi}{\partial V} \right] - \frac{\partial V}{\partial \psi} \langle \mathbf{j}_{ni} \cdot \mathbf{B} \rangle \tag{1}$$

We can then re-write this with $\psi$ or $\Phi$ or $\rho$ as independent variable:

### 2.1 $\Phi$

$\hat{\Phi} = \Phi/\Phi_b$, always [0,1]

$$\sigma_\parallel \left( \left. \frac{\partial \psi}{\partial t} \right|_{\hat{\Phi}} - \frac{\hat{\Phi}\dot{\Phi}_b}{\Phi_b} \frac{\partial \psi}{\partial \hat{\Phi}} \right) = \frac{F^2}{4\pi^2 \mu_0 \Phi_b^2} \frac{\partial}{\partial \hat{\Phi}} \left[ g_2 g_3 \frac{\partial \psi}{\partial \hat{\Phi}} \right] - \frac{1}{\Phi_b} \frac{\partial V}{\partial \hat{\Phi}} \langle \mathbf{j}_{ni} \cdot \mathbf{B} \rangle \tag{2}$$

Which is equal to E. Fable's notes.

### 2.2 $\rho$

$\rho = \sqrt{\Phi/\pi B_0}, d\Phi = 2\pi B_0 \rho d\rho$, and $\hat{\rho} = \sqrt{\hat{\Phi}} = \sqrt{\Phi/\Phi_b}, d\Phi = 2\Phi_b \hat{\rho} d\hat{\rho}$

$$\sigma_\parallel \left( \left. \frac{\partial \psi}{\partial t} \right|_\rho - \frac{\rho \dot{B}_0}{2B_0} \frac{\partial \psi}{\partial \rho} \right) = \frac{F^2}{16\pi^4 \mu_0 B_0^2 \rho} \frac{\partial}{\partial \rho} \left[ \frac{g_2 g_3}{\rho} \frac{\partial \psi}{\partial \rho} \right] - \frac{1}{2\pi\rho} \frac{\partial V}{\partial \rho} \frac{\langle \mathbf{j}_{ni} \cdot \mathbf{B} \rangle}{B_0} \tag{3}$$

Which is equal to [1] since $\frac{G_2}{J} = \frac{R_0}{16\pi^4} \frac{g_2 g_3}{\rho}$

$$\sigma_\parallel \left( \left. \frac{\partial \psi}{\partial t} \right|_{\hat{\rho}} - \frac{\hat{\rho}\dot{\Phi}_b}{2\Phi_b} \frac{\partial \psi}{\partial \hat{\rho}} \right) = \frac{F^2}{16\pi^2 \mu_0 \Phi_b^2 \hat{\rho}} \frac{\partial}{\partial \hat{\rho}} \left[ \frac{g_2 g_3}{\hat{\rho}} \frac{\partial \psi}{\partial \hat{\rho}} \right] - \frac{B_0}{2\Phi_b \hat{\rho}} \frac{\partial V}{\partial \hat{\rho}} \frac{\langle \mathbf{j}_{ni} \cdot \mathbf{B} \rangle}{B_0} \tag{4}$$

## 3 Thermal transport

$$V' \frac{\partial}{\partial t}[n_e T_e] = \frac{\pi B_0}{\Phi_b} \frac{\partial}{\partial \hat{\rho}} \frac{g_1 n_e}{V'} \chi_e \frac{\partial T_e}{\partial \hat{\rho}} + V' P_e \tag{5}$$

## 4 Finite Element forms

$$\sigma_\parallel \left( \left. \frac{\partial \psi}{\partial t} \right|_{\hat{\rho}} - \frac{\hat{\rho}\dot{\Phi}_b}{2\Phi_b} \frac{\partial \psi}{\partial \hat{\rho}} \right) = \frac{F^2}{16\pi^2 \mu_0 \Phi_b^2 \hat{\rho}} \frac{\partial}{\partial \hat{\rho}} \left[ \frac{g_2 g_3}{\hat{\rho}} \frac{\partial \psi}{\partial \hat{\rho}} \right] - \frac{B_0}{2\Phi_b \hat{\rho}} \frac{\partial V}{\partial \hat{\rho}} j_{ni} \tag{6}$$

is rewritten as

$$\frac{16\pi^2 \mu_0 \Phi_b^2 \hat{\rho} \sigma_\parallel}{F^2} \left. \frac{\partial \psi}{\partial t} \right|_{\hat{\rho}} = \frac{\partial}{\partial \hat{\rho}} \left[ \frac{g_2 g_3}{\hat{\rho}} \frac{\partial \psi}{\partial \hat{\rho}} \right] + \frac{16\pi^2 \mu_0 \Phi_b^2 \hat{\rho} \sigma_\parallel}{F^2} \frac{\hat{\rho}\dot{\Phi}_b}{2\Phi_b} \frac{\partial \psi}{\partial \hat{\rho}} - \frac{16\pi^2 \mu_0 \Phi_b}{2F^2} \frac{\partial V}{\partial \hat{\rho}} \langle \mathbf{j}_{ni} \cdot \mathbf{B} \rangle \tag{7}$$

or

$$m_\psi \frac{\partial \psi}{\partial t} = a_\psi \frac{\partial \psi}{\partial \hat{\rho}} + \frac{\partial}{\partial \hat{\rho}} d_\psi \frac{\partial \psi}{\partial \hat{\rho}} + s_\psi \tag{8}$$

with

$$m_\psi = 16\pi^2 \mu_0 \hat{\rho} \frac{\Phi_b^2 \sigma_\parallel}{F^2} \tag{9}$$

$$a_\psi = 16\pi^2 \mu_0 \hat{\rho}^2 \frac{\dot{\Phi}_b \Phi_b \sigma_\parallel}{2F^2} \tag{10}$$

$$d_\psi = \frac{g_2 g_3}{\hat{\rho}} \tag{11}$$

$$s_\psi = -8\pi^2 \mu_0 \frac{B_0 \Phi_b}{F^2} \frac{\partial V}{\partial \hat{\rho}} j_{ni} \tag{12}$$

$$\tag{13}$$

Now write $\psi$ as a sum of spatial basis functions

$$\psi(\rho, t) = \sum_{\alpha=1}^{n_{sp}} \Lambda_\alpha(\hat{\rho}) \hat{y}_\alpha(t) \tag{14}$$

then the weak form, after projection on $\Lambda_b$ and integration by parts is

$$\sum_{\alpha=1}^{n_{sp}} \frac{\mathrm{d}\hat{y}_\alpha(t)}{\mathrm{d}t} \int_0^1 m \Lambda_\beta \Lambda_\alpha \mathrm{d}\hat{\rho} = \sum_{\alpha=1}^{n_{sp}} \hat{y}_\alpha \int_0^1 a_\psi \Lambda_\beta \frac{\partial \Lambda_\alpha}{\partial \hat{\rho}} \mathrm{d}\hat{\rho} \tag{15}$$

$$- \sum_{\alpha=1}^{n_{sp}} \hat{y}_\alpha \int_0^1 d_\psi \frac{\partial \Lambda_\beta}{\partial \hat{\rho}} \frac{\partial \Lambda_\alpha}{\partial \hat{\rho}} \mathrm{d}\hat{\rho} + \left[ d_\psi \Lambda_\beta \frac{\partial \psi}{\partial \hat{\rho}} \right]_0^1 + \int_0^1 \Lambda_\beta s_\psi \mathrm{d}\hat{\rho} \tag{16}$$

which gives the matrix form

$$\mathbf{M}_\psi \frac{\mathrm{d}\hat{\psi}}{\mathrm{d}t} = (-\mathbf{D}_\psi + \mathbf{A}_\psi)\hat{\psi} + \mathbf{l} + \mathbf{s} \tag{17}$$

The boundary term $\mathbf{l}$ contains only the last element

$$\left. d_\psi \Lambda_\beta \frac{\partial \psi}{\partial \hat{\rho}} \right]_{\rho=1} = \left. \frac{g_2 g_3}{\hat{\rho}} \frac{\partial \psi}{\partial \hat{\rho}} \right]_{\rho=1} = \frac{16\pi^3 \mu_0 \Phi_b}{R_0 B_0} I_p \tag{18}$$

## 5 Other quantities

$$\iota = \frac{1}{q} = \frac{\partial\psi}{\partial\Phi} = \frac{\partial\rho}{\partial\Phi}\frac{\partial\hat{\rho}}{\partial\rho}\frac{\partial\psi}{\partial\hat{\rho}} = \frac{1}{2\Phi_b\hat{\rho}}\frac{\partial\psi}{\partial\hat{\rho}} \tag{19}$$

$$I_p = \frac{F}{8\pi^3\mu_0\Phi_b}\frac{g_2 g_3}{2\hat{\rho}}\frac{\partial\psi}{\partial\hat{\rho}} = \frac{g_2 g_3}{8\pi^3\mu_0}\frac{F}{q} \tag{20}$$

$$j_{tor} = \frac{R_0}{2\pi\mu_0}\frac{\partial}{\partial V}g_2\frac{\partial\psi}{\partial V} = \frac{R_0}{2\pi\mu_0\hat{V}'}\frac{\partial}{\partial\hat{\rho}}\frac{g_2}{\hat{V}'}\frac{\partial\psi}{\partial\hat{\rho}} \tag{21}$$

## References


[1] G. V. Pereverzev and P.N. Yushmanov. ASTRA Automated System for TRansport Analysis in a Tokamak. Technical Report 5/98, IPP Report, February 2002.